CHAPTER 4

THE SUBLIME

The birds of the air • The best theory in the world • No logical path to it • Electricity and magnetism • Their qualities • Their quantitative laws • Ampère's sympathetic understanding of the phenomena • Faraday's imaginary lines of force • Maxwell's factual field • Dodging the beasts of the field • Quantum dreams

> There is a delicate empiricism which makes itself utterly identical with the object, thereby becoming true theory. But this enhancement of our mental powers belongs to a highly evolved age.
>
> —Goethe, *Maxims and Reflections*

> One of the points I have laboured in this book is the unitary source of mystical and scientific modes of experience.
>
> —Arthur Koestler, *The Sleepwalkers*

> The ultimate goal would be: to grasp that everything in the realm of fact is already theory.
>
> —Goethe, *Maxims and Reflections*

THE BIRDS OF THE AIR

In his essay "Frogs and Birds," the introduction to a collection of papers by the Russian mathematician Y. Manin, Freeman Dyson wrote:

> Some mathematicians are birds, others are frogs. Birds fly high in the air and survey broad vistas of mathematics out to the far horizon. They delight in concepts that unify our thinking. . . . Frogs live in the mud below and see only the flowers that grow nearby. They delight in the details of particular objects, and they solve problems one at a time. I happen to be a frog, but many of my best friends are birds.

Dyson's metaphorical classification of mathematicians applies to physicists too. I was mostly a clumsy sort of frog physicist, elaborating or testing other people's ideas, but I was a bird manqué. I studied the work of birds, among them Newton, Ampère, Maxwell, Einstein, Schrödinger, Feynman, Gell-Mann, and Weinberg—men who had intuited and discovered wonderful, astonishing things about the world. Perhaps I could do it too. Deep inside, every physicist dreams of glory and believes it is attainable, or once did so; otherwise he or she wouldn't be in the field.

As long as I was immersed in doing physics, the discovery of its laws seemed a natural and obvious process. Now when I look at the field and realize that I can no longer recall the origin or proof of some facts I once knew as second nature, I am awed that anyone was ever able to penetrate through the phenomena to the laws. In a speech on the principles of research, given in 1918 in honor of Max Planck, the discoverer of the quantum, Einstein captured the mystery of the birds' accomplishments: "There is no logical path to these laws; only intuition, resting on sympathetic understanding of experience, can reach them."

Of all theories, the best in the world is quantum electrodynamics, appropriately called QED. QED is the quantum theory of the electron and its interaction with light, and it determines just about everything relevant to the physics and chemistry of the atoms and molecules that compose us and the world around us. We trust it because it predicts the values of measured properties of the atom so precisely as to strain belief. Any successful theory in physics is amazing; QED is a miracle.

As I'll show in the next chapter, the creation of QED runs in counterpoint to the development of the Efficient Market Model in finance. The development of QED is a tight intertwining of data, facts, experiments, failures, and successes; the development of the Efficient Market Model is coupled to the world much more loosely, driven as much by ideology as by facts. QED predicts atomic properties to an accuracy of more than 10 significant figures; the best models in finance are not accurate to even one. More to the point, no one in finance knows how to specify exactly what "accurate" means, because so many of its variables are related to human sentiment.

In this chapter I want to provide a glimpse of the crooked paths that culminate in theories, to give a sense of the role of intuition in the discovery of a theory and in the sweep of that discovery. I want to recount in stylized fashion some of the prodigious feats that birds and frogs have achieved as they created the best theory in the world. Here, then, is an attenuated trip through the history of classical and quantum electromagnetic theory, a triumph of mind over matter.

THE PHENOMENA: ELECTRICITY AND MAGNETISM

First the facts. The ancient Greeks knew that rubbing amber, the fossilized tree resin called *elektron* in Greek, empowered it to pick up tiny scraps of straw or feather. We now call this *static* electricity, the buildup of a stationary electric charge on the surface of objects, a terminology that prefigures the later discovery that what we call an electric current is the *dynamic* flow of charge. The Greeks were also familiar with lodestones, naturally occurring magnets that they discovered in Magnesia in the province of Thessaly.

Then the theories. Lodestones pointed north, but why? Some thought the attractor was the polestar in the heavens; others imagined a small magnetic island located at the north pole itself. Then, in 1600, William Gilbert, an English physician, proposed that the Earth itself is a giant magnet with an iron core. Proof was his experiment on the angles at which freely suspended needles "dip" in the vicinity of a spherical lodestone, which he noticed was similar to the way compass needles incline to the surface of the Earth. His theory is now a fact.

QUALITIES: POSITIVE AND NEGATIVE

Yet electromagnetic theory was slow to take flight. More than 50 years after Newton began applying the calculus to the motion of the planets, at about the same time (1738) that Daniel Bernoulli published his seminal book on hydrodynamics, the theory of electricity and magnetism was just beginning to incorporate facts.

That there are two kinds of charge was a discovery made by the French chemist Charles Du Fay in 1734. He found that he could create two different types of electricity by friction: *vitreous* by rubbing glass-like materials, and *resinous* by rubbing resin-like materials. Furthermore the small pieces of charged glass that repelled each other attracted pieces of charged amber. Some years later Benjamin Franklin replaced Du Fay's empirically inspired adjectives by the moral descriptors *positive* and *negative*.

Note that electrical charge didn't *have to* come with a positive or negative sign. In gravitation everything is positively charged and masses never repel each other. The electron's antiparticle, even though it has the opposite electric charge, has the same gravitational mass. Masses carry no + or −.

QUANTITIES: COULOMB'S LAW OF FORCE BETWEEN STATIC CHARGES

In 1788, inspired by Newton's law of gravitation, Charles Coulomb published what is now called Coulomb's law: the attractive or repulsive force between small point-like charges is proportional to the magnitude of their charge and diminishes in proportion to the inverse square of the distance between them. But take no discovery for granted: though it sounds simple, it took ingenious experimentation to accurately measure both the quantity of charge on objects and the forces between them.

No one thought much about the transmission of forces. People believed, even took for granted, that both the gravitational force and the electrostatic force were transmitted through space instantaneously by so-called action at a distance.

VOLTA'S ITALIAN INSIGHT: CHEMISTRY IS BETTER THAN FRICTION

Facts accumulate. Until this moment the only way to produce electric charge was by means of friction, scraping charge off objects and storing it in small quantities in a reservoir, a Leyden jar for example. Electric currents fueled by such capacitors were consequently brief and transient. Then, in 1800, Alessandro Volta invented the first chemical source of electric charge, the *voltaic pile*, a battery consisting of alternating disks of copper and zinc separated by cardboard soaked in salt water.

It wasn't immediately obvious to anyone that what flowed out of the battery was electric charge, but its effects were soon identified with those caused by the brief flow of charge produced the old-fashioned, frictional way. Large voltaic piles produced powerful, steady, long-lived currents suitable for experimenting with. The discovery that running a current through water separated it into its component parts, hydrogen and oxygen, soon followed. Thus electrochemistry was born.

OERSTED: ELECTRIC CURRENTS BEHAVE LIKE MAGNETS

More observations. Until now electricity and magnetism had seemed to be two disparate phenomena. Then, in 1820, Hans Christian Oersted discovered a crossing of boundaries: he observed that electric currents could deflect compass needles.

AMPÈRE: A LAW FOR THE FORCE BETWEEN CURRENTS

If electric currents deflect compass needles, then currents behave like magnets, and must therefore interact with other magnet-like currents. Hearing of Oersted's discovery, André-Marie Ampère immediately began his own series of much more thorough and quantitative investigations.

First, he found that pairs of electric currents moving in the same direction attract, whereas pairs moving in opposite directions repel, as shown in Figure 4.1. Then, after using a voltaic pile to drive cur-

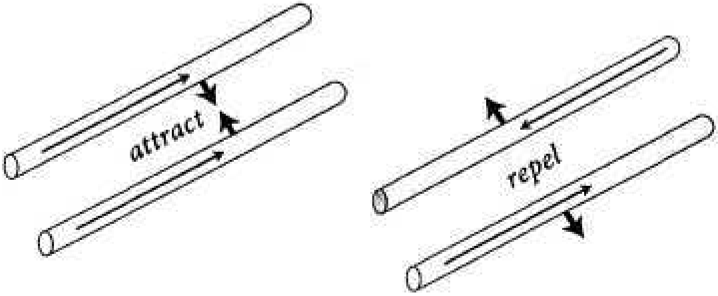


Figure 4.1. Ampère: Currents moving in the same direction attract. Currents moving in opposite directions repel.

rent through a helical loop of wire, he noticed that the force exerted on a compass needle by the loop was much like the force exerted by a magnet. He concluded that magnets themselves might be small solenoidal currents (see Figure 4.2)

This was qualitative, and relatively simple. Ampère's theoretical tour de force was his discovery of the mathematical formula for the magnetic force between two isolated *current elements*, tiny imaginary bits of current that are the magnetic analogues of point-like electric charges. I say "imaginary" because it is impossible to isolate small steady current elements. Currents from batteries can flow only in finite closed loops from one terminal to another.

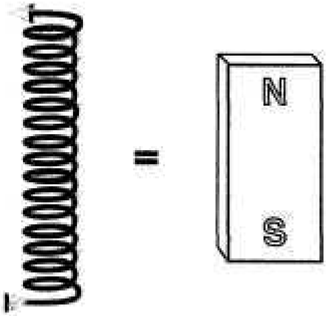
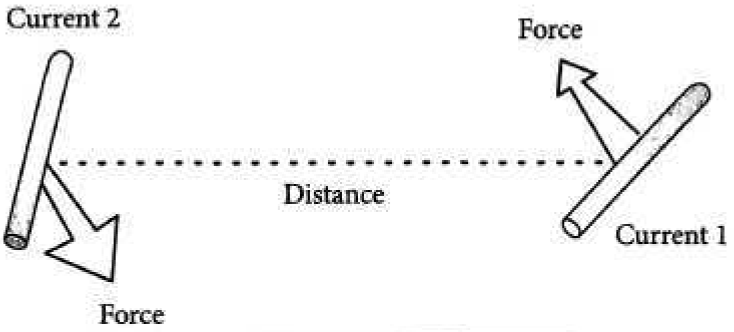


Figure 4.2. Ampère: Small current loops behave like little north-south bar magnets.

Ampère's law for the force between two imaginary bits of wire, as illustrated in Figure 4.3, was vastly more complex and subtle than the Coulomb force between two point charges, because charges have location but no orientation, whereas currents necessarily point in some direction. Ampère's force therefore varied not only with distance between the current elements but also with the orientation of each element, and is much more difficult to describe or visualize. Its discovery was a triumph of imagination and intuition.

Using his law Ampère was able to mathematically sum up the forces from each infinitesimal segment of current to determine the forces between entire circuits. His theory shifted the foundations of electricity and magnetism, transforming the field from a largely descriptive subject into a quantitative discipline in which one could employ the calculus to tackle realistic engineering problems.



$$\boxed{\frac{I_1 d\vec{\ell}_1 \times (I_2 d\vec{\ell}_2 \times \hat{r}_{21})}{|\mathbf{r}_{21}|^2}}$$

Figure 4.3. Ampère's Law: The magnetic forces that two current elements, I_1 of length dl_1 and I_2 of length dl_2, separated by a distance r_{21}, exert on each other. Each element can point in an arbitrary direction, and they can be separated by an arbitrary distance. Ampère's formula is shown boxed.

A SYMPATHETIC UNDERSTANDING

Ampère titled his paper "Theory of Electrodynamic Phenomena, Uniquely Deduced from Experience." But as Henri Poincaré remarked in 1905 about "Ampère's immortal work," Ampère's laws could *not* have been deduced from experience, because he had no infinitesimal currents to experiment with. Only what Einstein called intuition or a "sympathetic understanding of experience" could have led him from observations of entire circuits to a law for infinitesimal current elements. In his encyclopedic *A Treatise on Electricity and Magnetism*, James Clerk Maxwell later wrote:

> The experimental investigation by which Ampère established the laws of the mechanical action between electric currents is one of the most brilliant achievements in science.
>
> The whole, theory and experiment, seems as if it had leaped full grown and full armed from the brain of the "Newton of Electricity." It is perfect in form and unassailable in accuracy, and it is summed up in a formula from which all the phenomena may be deduced, and which must always remain the cardinal formula of electro-dynamics.
>
> The method of Ampère, however, though cast into an inductive form, does not allow us to trace the formation of the ideas which guided it. We can scarcely believe that Ampère really discovered the law of action by means of the experiments which he describes. We are led to suspect, what, indeed, he tells us himself, that he discovered the law by some process which he has not shown us, and that when he had afterwards built up a perfect demonstration, he removed all traces of the scaffolding by which he had built it.

Maxwell, who performed his own magic, recognized that miraculous discoveries seem to leap out of the mind's invisible Dirac sea, elicited by intuition.

FARADAY: MOVING MAGNETS CREATE ELECTRIC CURRENTS

Back to the phenomena. Michael Faraday, impressed by Oersted's discovery that one current could push on another, became convinced that one current alone should be able to *produce* another current too. In 1831 he ran a *steady* current through one circuit and watched for a steady current in another. Disappointed at observing nothing, he switched off the current in the first circuit—and unexpectedly observed a flicker of current in the second. The *change of current* in the first wire induced an electric current in the second.

Because he knew that currents behave like magnets, Faraday conjectured and then demonstrated that moving a magnet near a closed loop of wire could induce an electric current in it too, as illustrated in Figure 4.4. Well acquainted with the force that magnets exert on

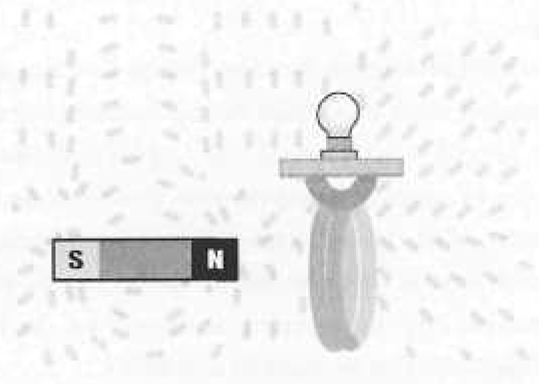


Figure 4.4. Faraday's discovery of induction: a moving magnet induces an electric current in a circuit.

currents, Faraday quickly figured out how to build the first electric motor by surrounding a loop of current-carrying wire with magnets, which made it rotate. Then he did the inverse: by rotating a loop of wire in the vicinity of a magnet he produced a current in the loop and created the first electric generator. The discovery of phenomena and the development of theory have played leapfrog in the history of electromagnetism, a fruitful progression missing in the development of models of financial markets.

FARADAY IMAGINES FORCE-TRANSMITTING LINES

Until Faraday everyone assumed that charges and currents pushed on each other across empty space without any delay. All interactions were thought to reside purely within the objects that felt them. Faraday knew little mathematics beyond algebra, but he was adept at thinking spatially. In a Spinozan act of identification with the object, Faraday dreamed up *lines of force*. He pictured every charge exuding *electric lines of force* that flowed though space to create an invisible web, transmitting and exerting electric forces on charges in other locations. He visualized *magnetic lines of force* similarly pervading space to attract iron filings and to influence distant currents. Figure 4.5 shows the schematic lines of electric and magnetic forces necessary to account for the distant electrical influences of charges and magnets.

Faraday recast all the relationships between electric and magnetic forces known at that time in terms of equivalent dynamic relationships between the lines, so that the laws of electromagnetic interactions became laws of interacting lines. Thus, for example, his own discovery that "magnets in motion induce electric currents in circuits" was transformed into the more abstract proposition "A change in the number of magnetic lines of force creates electric lines of force."

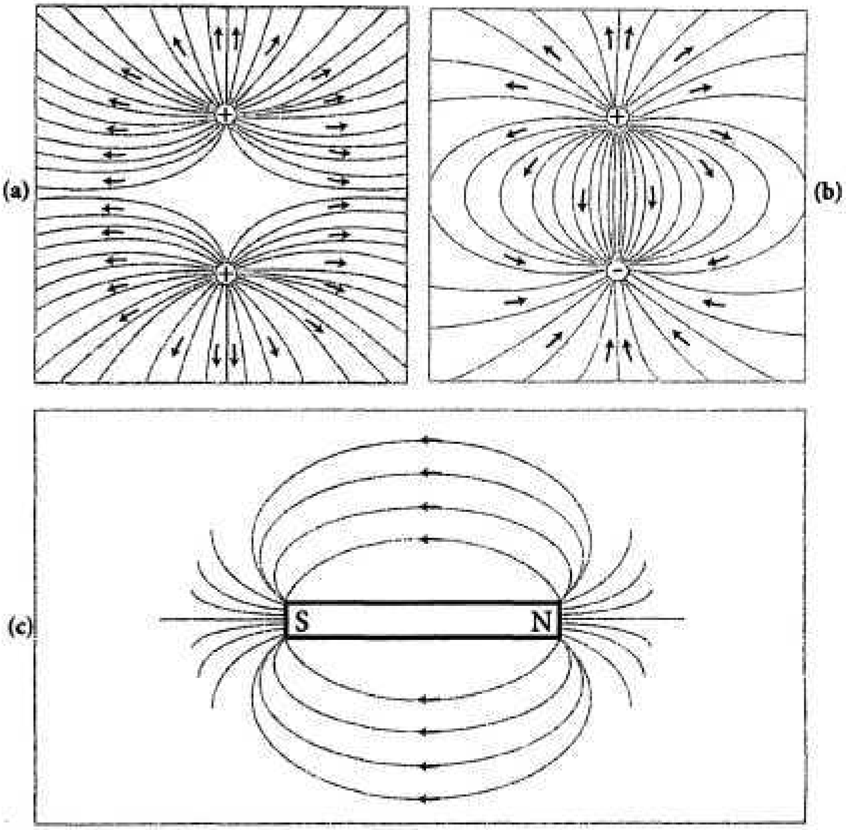


Figure 4.5 **(a)** Electric lines of force from two positive charges. **(b)** Electric lines of force from one positive and one negative charge. **(c)** Magnetic lines of force emanating from a bar magnet.

In a paper he published in 1846, almost 60 years ahead of its time, Faraday presciently began to regard electromagnetic radiation as existing in its own right, without the need for a medium to flow through:

> The view which I am so bold to put forth considers, therefore, radiation as a kind of species of vibration in the lines of force which are known to connect particles and also masses of matter together. It endeavors to dismiss the aether, but not the vibration.

The lines originated as metaphor; Faraday began to liberate them; shortly thereafter, Maxwell set them free.

MAXWELL MODELS THE LINES

> From a long view of the history of mankind—seen from, say, ten thousand years from now—there can be little doubt that the most significant event of the 19th century will be judged as Maxwell's discovery of the laws of electrodynamics. The American Civil War will pale into provincial insignificance in comparison with this important scientific event of the same decade.
>
> —Richard Feynman, *Lectures on Physics*

The historic achievement of James Clerk Maxwell, a Scottish mathematician and theoretical physicist with a very practical streak, was the unification of electricity and magnetism into a consistent set of equations for electromagnetic theory. Feynman, no mean achiever himself, is accurate about the magnitude of Maxwell's discovery, which was the midpoint on the trajectory from Newton's discovery of the laws of mechanics to Einstein's theories of relativity. Maxwell was a bird who flew from branch to branch and field to field, not only developing electromagnetic theory but also playing a major role in the creation of thermodynamics, statistical mechanics, and the theory of controlling engineering devices.[1]

Maxwell made his first assault on electromagnetism in 1856, in a paper entitled "On Faraday's Lines of Force." Faraday's imaginary electric and magnetic lines of force reminded Maxwell of the streamlines in the flow of a fluid. By the mid-1850s the theory of fluid flow, or hydrodynamics, was mathematically sophisticated and powerful. Maxwell, inspired by Faraday's vision, decided to model Faraday's lines of force as "the motion of an imaginary fluid," hoping to get some insight from what he thought of as only an analogy rather than reality. It was a warm-up exercise for Maxwell, who clearly understood the difference between a theory and a model. Working by analogy, he explicitly hoped to "avoid the dangers arising from a premature theory professing to explain the cause of the phenomena." Maxwell's tentative model for electromagnetism was based on the better-understood theory of fluid flow and was meant to suffice until "a mature theory, in which the physical facts will be physically explained, will be formed." The result was a set of differential equations for the interactions between the still imaginary fluid lines of electric and magnetic force filling space.

MAXWELL REIFIES THE LINES

Until now Maxwell had merely translated Faraday's intuition about imaginary lines of force into the more formal language of differential equations, using analogies with fluid flow as an aid to thinking. Then he crossed the threshold: in 1861, in a paper entitled "On Physical Lines of Force," he began to regard the lines as genuine stresses in a space-filling ether. In Maxwell's hands, Coulomb's law, Ampère's laws, and Faraday's law all became propositions about the dynamics of lines of force. Here are two examples of how the laws of forces on visible objects were rewritten as laws of invisibly interacting *lines of force*:

1. Ampère's law of force specified how two current elements pushed against each other. It was a statement about observable effects. Rephrased by Maxwell it became:

   Ampère's law: An electric current creates *magnetic* lines of force.

In Maxwell's view, one electric current element produces magnetic lines of force that permeate the space around it. These lines, like a magnet, push on the other current element.

2. Faraday's law of induction stated that the fluctuation of an electric current in one circuit induces a current in another. Rephrased by Maxwell, it stated:

   Faraday's law: A change in the *magnetic* lines of force creates *electric* lines of force.

   A fluctuating electric current in the first circuit produces a corresponding fluctuation in the number of magnetic lines of force that fill the space around it. This fluctuation in turn creates electric lines of force that impel the charges in the second circuit to flow; hence an induced current. Charges and current no longer act on each other directly. Now their actions are mediated by the lines.

MAXWELL MODIFIES AMPÈRE'S EQUATIONS

There is a glaring asymmetry between Ampère's law and Faraday's law. Whereas Faraday's law states that electric lines of force are created by a change in magnetic lines, Ampère's law says that magnetic lines are created only by electric currents. Why shouldn't changes in electric lines similarly induce magnetic lines? Guided by intuition and a sense of symmetry, Maxwell added an entirely new law for the production of lines of force, a law obtained by switching the words *magnetic* and *electric* in Faraday's law in (2) above, transforming it into:

3. *Maxwell's addition to Ampère's law:* A change in the *electric* lines of force creates *magnetic* lines of force.

Experiment didn't demand this law, and historians of science still disagree about the impetus that led Maxwell to make the addition. But what matters most is that, without experimental evidence, he perceived the necessary existence of a phenomenon that hadn't yet been observed.

MAXWELL'S THEORY: THE FIELD ITSELF

> Since Maxwell's time, physical reality has been thought of as represented by continuous fields, and not capable of any mechanical interpretation. This change in the conception of reality is the most profound and the most fruitful that physics has experienced since the time of Newton.
>
> —Albert Einstein, "Maxwell's Influence on the Development of the Conception of Physical Reality," in *James Clerk Maxwell: A Commemorative Volume 1831–1931* (1931), 71.

Why the extreme praise? Because Maxwell changed the way physicists do physics. He examined the equations obeyed by the visible world, saw a pattern with something missing, completed it, and deduced the existence of electromagnetic waves. The imposition of an apparently missing symmetry has become the classic modus operandi of theoretical physics. The discovery of the Dirac equation, the unveiling of quarks, and the elucidation of the Standard Model all proceeded in similar style.

In 1864 Maxwell published "A Dynamical Theory of the Electromagnetic Field." In this paper he christened the lines of force *the field*, writing, "The electromagnetic field is that part of space which contains and surrounds bodies in electric or magnetic conditions." The lines of force, once an aid to thinking, became a theory.

I like the phrase "a dynamical theory." Maxwell's equations describe interactive movement, the shapes of the electric and magnetic fields as they twist and curl through space and influence each other. Like most things and ideas, the field is observable only indirectly, via its effects, but no one doubts its reality.

MAXWELL'S EQUATIONS: THE FIELD'S GEOMETRY—CURLS AND DIVERGENCES

As shown in Figure 4.6, a field in general can diverge like a fountain or curl like a halo.[2] Maxwell's equations elegantly describe the causes of the electromagnetic field's curls and divergences, which can be understood pictorially.

I shall denote the electric field by the symbol $\vec{E}$ and the color black, and the magnetic field by the symbol $\vec{B}$ and the color gray. The arrow $\vec{\ }$ above each field's symbol indicates that it is a vector, meaning that at each point in space the field has not only a magnitude but also a direction, like a force, or a fluid flowing. Maxwell's

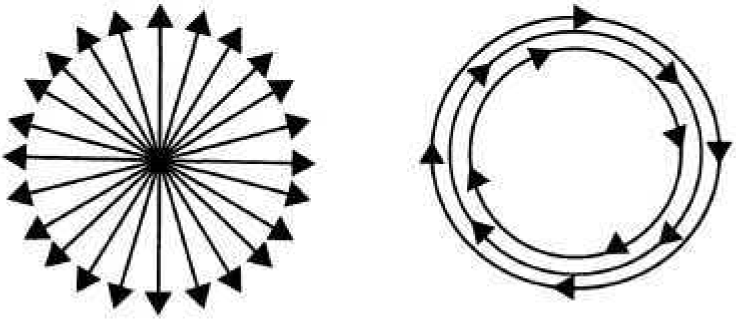


Figure 4.6. The configurations of fields: divergences and curls.

four famous equations concisely specify what causes $\vec{E}$ and $\vec{B}$ to diverge or curl, as illustrated in Figures 4.7–4.10.

To summarize, electric charges produce divergent $\vec{E}$ fields; electric currents produce curling $\vec{B}$ fields; time-varying $\vec{E}$ fields produce curling $\vec{B}$ fields; time-varying $\vec{B}$ fields produce curling $\vec{E}$ fields.

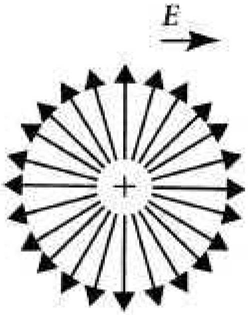


Figure 4.7. Maxwell's first equation: A positive electric charge is the source of a divergent electric field $\vec{E}$. Similarly a negative charge is the sink of a convergent $\vec{E}$ field. The number of electric lines of force is proportional to the charge.

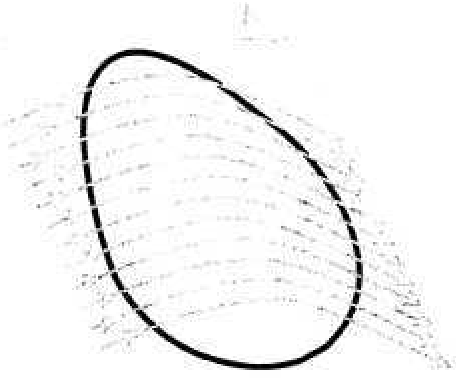


Figure 4.8. Maxwell's second equation: Magnetic fields $\vec{B}$ never diverge or converge, because there are no single magnetic poles found in nature to serve as their source.[3] Thus every line that flows *into* a closed surface must flow *out* again.

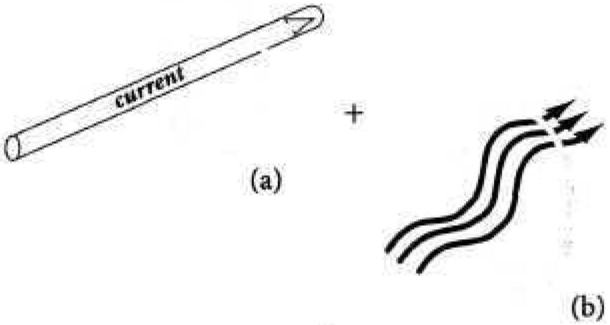


Figure 4.9. Maxwell's third equation: Since $\vec{B}$ lines cannot diverge, they can only curl. There are two independent ways to generate a curling magnetic field : (a) via a steady electric current (corresponding to Ampère's law), and (b) via a time-varying electric field (corresponding to Maxwell's modification of Ampère's law).

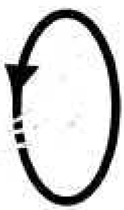


Figure 4.10. Maxwell's fourth equation: Electric fields can curl as well as diverge. Fluctuating magnetic fields produce curling electric fields (corresponding to Faraday's law).

THE GREAT CONFIRMATION: LIGHT IS THE PROPAGATION OF ELECTROMAGNETIC WAVES

How did Maxwell know that his addition to Ampère's law was correct? Because that addition made it possible for waves of $\vec{E}$ and $\vec{B}$ fields to propagate indefinitely through empty space.

By combining the equations depicted in Figures 4.9a, 4.9b, and 4.10, you can see how waves can be produced and then propagate, as illustrated in Figure 4.11. It begins with an *oscillating* electric current, which, by Figure 4.9a, generates an *oscillating* curling magnetic field. The oscillation in time is crucial, because it is the magnetic field's oscil-

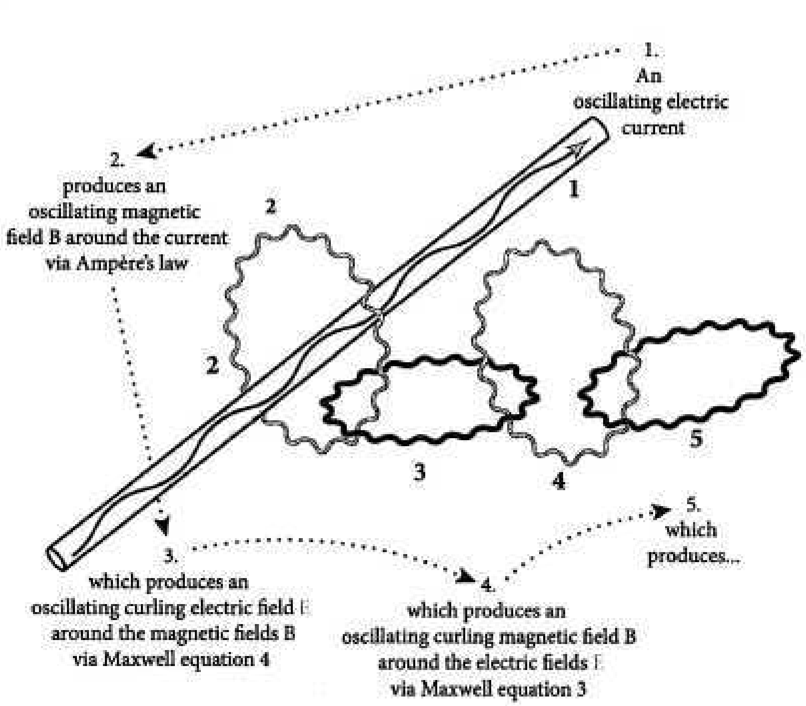


Figure 4.11. The generation and propagation of electromagnetic waves via Maxwell's equations.

Step 1: An oscillating electric current in the wire (an antenna)
Step 2: Produces an oscillating magnetic field via Ampère's law;
Step 3: Which, via Maxwell equation 4, produces a **curling oscillating electric field.**
Step 4: The extra term in Maxwell's equation 3 guarantees that the **oscillating electric field** will induce a *fresh* oscillating curling magnetic field.
Step 5: Which produces a **curling oscillating . . .**

The diagram is meant to be schematic. Only parts of the field are shown. The wiggles in the loops are meant to indicate the oscillatory nature of the fields.

lations, according to Figure 4.10, that induce a similarly oscillating curling electric field, which in turn, according to Figure 4.9b, induces another oscillating curling magnetic field, which . . . This is, precisely, a wave! Waves of electric and magnetic fields propagate through space by leapfrogging, an oscillating $\vec{E}$ leading to an oscillating $\vec{B}$ leading to an oscillating $\vec{E}$ ad infinitum. You can think of the current-bearing wire in the center of the figure as an antenna that initiates the radiation, which then continues to re-create itself recursively.

Working out the mathematics of the leapfrogging, Maxwell discovered that the theoretical speed with which the waves propagated through space was about 186,000 miles per second, the measured speed of light. Therefore, he deduced, the light we see is in fact an electromagnetic wave whose frequencies happen to be visible to our eyes.[4] Heat, ultraviolet rays, radio waves, and X-rays are merely the same Maxwellian fields, differing from each other only in their wavelength. Without Maxwell's addition to the equations, there would have been no leapfrogging and no waves in the theory at all.

One can learn even more about the waves of light by examining Figure 4.12. There the black and gray loops give rise to each other sequentially, like a father and child building a tower of hands. Each $\vec{E}$ wave curls about the previous $\vec{B}$ wave, and each $\vec{B}$ wave curls about the previous $\vec{E}$ wave, so that at any point in space the electric and magnetic fields curl perpendicular to each other, and also perpendicular to the direction in which they move. That is, the waves propagate transversely, as illustrated. In this way Maxwell's equations explained the fact that light can be polarized. The transverse propagation of light is quite different from the so-called longitudinal propagation of sound waves, in which the variations in air pressure occur along the direction in which the wave travels.

In 1887 Heinrich Hertz used an antenna to create, transmit, receive, and reflect electromagnetic waves in his laboratory, entirely confirming the theory. About Maxwell's equations he later said:

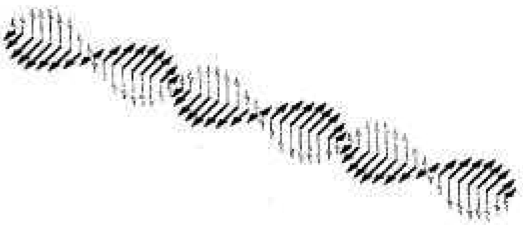


Figure 4.12. The transverse propagation of electromagnetic waves.

> One cannot escape the feeling that these mathematical formulae have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers, that we get more out of them than was originally put into them.[5]

REALITY = PERFECTION; FACT = THEORY

There is no better or truer description of the physical phenomenon of light than to say it *is* precisely the electromagnetic field defined by Maxwell's equations. The electromagnetic field is not *like* Maxwell's equations; it *is* Maxwell's equations. While a model builder knows that his model airplane is *like* a true airplane,[6] and a climate modeler is aware that his equations only simulate the atmosphere, a physicist knows that light and Maxwell's equations are one and the same.

I see the trajectory of the discoveries of electromagnetism as follows:

From phenomena as facts
    to lines of force as an aide to visualization
        to analogies with fluid flow
            to a theory of fields that has again become fact.

We advance by mounting new theories atop previous facts. If those theories prove correct, they become facts too.

THE BEASTS OF THE FIELD

> Who teaches us more than the beasts of the field and makes us wiser than the birds of the heavens?
>
> —Job 35:11

The advances of the sublime theory of electromagnetism didn't end in the late nineteenth century. In 1897 J. J. Thomson discovered the electron, the minuscule carrier of the electric current. In 1900 Max Planck noticed the first hint of the quantum nature of light, and in short order the completeness of classical physics—Newton for matter and Maxwell for light—began to unravel. Einstein showed that smooth light waves consist of chunky quanta, and Schrödinger that chunky electrons satisfy smooth quantum mechanical wave equations.

The *idea* of the electromagnetic field is Maxwell's equations. The *idea* of the electron is the Dirac equation. Combining them both we obtain QED, the *idea* of electrons interacting with light quanta, the theory of relativistic *quantum electrodynamics*. The equations that compose it, Maxwell's and Dirac's, were complete by the late 1920s.

QED seemed to work magnificently, the result of calculations using the theory agreeing perfectly with experiment. But in the late 1940s atomic physicists, among them Polykarp Kusch and Willis Lamb at Columbia, discovered tiny but very precise anomalies in the radiation emitted by the single electron orbiting the proton in a hydrogen atom.[7] The wavelengths they measured didn't quite match those predicted by the straightforward use of QED. Theorists set about trying to explain the size of the discrepancies and were led to consider the possibility that they might stem from the interactions of the atom with the invisible Dirac sea itself.

From a theoretical point of view, what we call a single isolated electron is neither single nor isolated: it travels through the Dirac sea, which is filled to the brim with an infinite number of invisible negative-energy electrons. When you accept the existence of the sea as gospel rather than metaphor, the Dirac theory of a supposedly single electron is actually a theory of *many* particles. There *is* no solitary electron; the sea of electrons is the medium in which all particles live. Electrons can pop out of it and leave behind empty positron-holes into which other electrons can then sink in temporary oblivion.

This medium influences everything. The simple Coulomb attraction between a hydrogen atom's proton and its orbiting electron is altered by their immersion in the sea. The sea's invisible electrons repel the orbiting electron; the repulsion modifies the orbit and hence alters the frequencies of the light waves it emits when it transitions from one orbit to another.[8] Keeping careful track of this complexity—the interactions of the electron with the vast sea and with its own electromagnetic field—is immensely difficult, both conceptually and technically.

Conceptual Difficulties: The Stand-Alone Electron Is Bare, the Real One Dressed

The real electron we see is not the stand-alone electron the Dirac equation begins with, undisturbed and private. Physicists call that hypothetical stand-alone electron the *bare* electron. The real electron, the only electron we can experiment on, suffers the drag of the sea; physicists call the real electron the *dressed* electron, because it is clothed by the bits of the sea that stick to it. The idea of *dressing* is a theoretical construct to take us from the imperfect unreal stand-alone electron to the perfect real one that exists.

Because it is clothed, the dressed electron moves differently than it would if there were no sea. To accurately calculate the properties of a real electron in a real hydrogen atom, one must deal with the motion of a dressed electron.

Technical Difficulties: Calculations Produce Infinities . . .

The drag of the sea changes the electron's resistance to motion, not unlike the way a car feels more massive if you drive it with the hand brake on. Because of the sea, the mass of the dressed electron (colloquially called the "dressed mass") differs from the mass of the bare electron (the "bare mass").

Similarly the perceived charge of a real electron, the dressed charge, differs from its bare charge because the negative-energy electrons in the sea surround and shield the bare electron, modifying the charge it displays to the outside world. The dressed electron is a bare electron immersed in a medium. Just as light travels more slowly through glass, so the electron behaves a little differently as it moves through the sea.

You would think that these sea changes should be small, because electromagnetic forces in general are relatively weak. But you can calculate their size in the full theory of QED that takes account of the medium, and—*the horror!*—they turn out to be infinite. The numerical difference between the dressed mass and the bare mass is infinity. Equally unfortunate and equally infinite is the difference between the dressed charge and the bare charge. These are the true beasts of the field.

. . . But Physicists Figure Out How to Evade Them

Formal mathematics can help physics, but has never been allowed to hinder it. New developments in mathematics, from calculus to topology, have often been initiated by physicists who, by means of intuition and persistence, have sneakily but sloppily invented new kinds of mathematics that were only later made rigorous by purists. Newton invented the calculus in the seventeenth century to handle mechanics, and its foundations were satisfactorily cleaned up years later by Augustin-Louis Cauchy and his contemporaries. In the late 1940s, reconnoitering around the technical difficulties of the Dirac sea, Richard Feynman, Julian Schwinger, and Shin'ichirō Tomonaga found an ingenious way to suppress the technical infinities of quantum electrodynamics by means of a judicious combination of extreme care and chicanery. Their starting point was never to forget that the *normal* quotidian electron we "see" every day is not the bare electron. The normal electron is the *fully dressed electron.* Therefore, when we look at an electron, with our eyes or our apparatus, what we see—what we know as fact—is its dressed mass and charge.

Consequently there is no point in using the theory to calculate corrections to the value of the mass and charge, since including them would be the *double counting* of an effect that is already there. But that doesn't mean that the effects of the sea don't matter. They do, *but not for the purposes of calculating the mass and charge of the electron,* since these we already know from measurement. We should instead use the theory only to calculate the *incremental* effects of the sea on the motion of an electron in the hydrogen atom, over and above its effects on the mass and charge.

Now for the Chicanery

The fact is that we don't care what the Dirac sea does to free electrons, because we have already accounted for the drag of the sea on the free electron by making use of the observed values of its mass and charge. All so-called free electrons are already conceptually dressed by the sea. Given that fact, what is important is the drag on the bound electron *over and above* the drag on the free electron. That's the effect we can actually observe.

What Feynman, Schwinger, and Tomonaga noticed, mirabile dictu, was that though the drag on both bound and free electrons was infinite, the *difference* between them was finite! All the infinities can be dodged if we calculate only the difference between the drag on the free and the drag on the bound electron. And that is the only difference that is observable. If differences are all we calculate, then nothing in any QED calculation is infinite.

The New Normal

Physicists call this process *renormalization*, a recalibrating of normality to accommodate the recognition that the "normal" reality and perfection of the world correspond not to what goes *into* the theory, but only to what comes *out* of it when you have solved it. The normal isolated free electron we see is the electron dressed by the sea, not the bare stand-alone Dirac electron. One must redefine "normal" to be the final reality, not the original imperfection.

Feynman, Schwinger, and Tomonaga painstakingly calculated the effects of the Dirac sea on the orbiting electron inside a hydrogen atom relative to the new normal. They found that the sea produced minute changes in the frequencies of light waves emitted by hydrogen, and that the calculations agreed with then contemporary experiments to the stunning accuracy of one part in several thousand. Nowadays the theory's predictions are accurate to one part in 100 billion or so, or, as Feynman put it, equivalent to measuring the distance from San Francisco to New York with the accuracy of a hairbreadth. Renormalization works unimaginably well.

The pragmatic and cunning lesson it teaches is this: use your theories to calculate only what you can really measure, and carefully avert your eyes from everything else.

> Nature will reveal nothing under torture; its frank answer to an honest question is "Yes! Yes!—No! No!"
>
> —Goethe, *Maxims and Reflections*

Financial modelers use a process similar to renormalization to force their less than perfect, less than real models to fit the world they observe. They call this process *calibration*, the tuning of parameters in a model until it agrees with the observable prices of liquid (i.e., easily tradable) securities whose values we know. Only when a model is forced to be consistent with this "normal" state of markets can we reasonably use it to calculate the value of "abnormal" securities whose prices we don't know. But calibration in finance works much less well than renormalization in physics: in physics the normal and abnormal are governed by the same laws, whereas in markets the normal is normal only while people behave conventionally. In crises the behavior of people changes and normal models fail. While quantum electrodynamics is a genuine theory of all reality, financial models are only mediocre metaphors for a part of it.

ELECTROMAGNETISM AS METAPHOR

Like all underlying facts and theories, electromagnetism serves as a metaphor with which to understand ourselves and the world around us, as pointed out by Owen Barfield in his book *History in English Words*:

> The phrase "high tension," used of the relation between human beings, is a metaphor taken from the condition of the space between two electrically charged bodies. At present many people who use such a phrase are still half-aware of its full meaning, but many years hence everybody may be using it to describe their quarrels and their nerves

> without dreaming that it conceals an electrical metaphor—just as we ourselves speak of a man's "disposition" without at all knowing that the reference is to astrology. . . .
>
> The scientists who discovered the forces of electricity actually made it possible for the human beings who came after them to have a slightly different idea, a slightly fuller consciousness of their relationship with one another. They made it possible for them to speak of the "high tension" between them. So that the discovery of electricity, besides introducing several new words (e.g., electricity itself) into our everyday vocabulary, has altered or added to the meaning of many older words, such as battery, broadcast, button, conductor, current, force, magnet, potential, tension, terminal, wire, and many others.

QUANTUM DREAMS

Once, after long and sustained efforts with the theory of quantum mechanics in graduate school, I had an exhausting but exhilarating dream: my entire body was a Schrödinger wave that *had to* satisfy Schrödinger's famous wave equation. In my dream I struggled to flex my body so as to make it undulate like a three-dimensional violin string, contorting myself to ensure that I satisfied the boundary conditions of the differential equation, oscillating inside the box I dream-inhabited, but pinned down at its walls.

I like to imagine that it was by some similar visceral but deeper intuition that the discoverers of electromagnetic theory and QED merged with their subject.

EPILOGUE

Everything that we call Invention or Discovery in the higher sense of the word is the serious exercise and activity of an original feeling for truth, which, after a long course of silent cultivation, suddenly flashes out into fruitful knowledge. It is a revelation working from within on the outer world, and lets a man feel that he is made in the image of God. It is a synthesis of World and Mind, giving the most blessed assurance of the eternal harmony of things.

—Goethe, *Maxims and Reflections*