CHAPTER 2

METAPHORS, MODELS, AND THEORIES

Language is a tower of metaphors • The hole in the Dirac sea • Metaphors become real: the discovery of the positron • Absence is a presence • Analytic continuation • Every fact is a theory • Building a model airplane • Why is a model a model? • Why is a theory a theory? • A puzzling case of monocular diplopia • Making the unconscious conscious again

THE DIRAC SEA

The Metaphorical Rests on the Physical

Sleep is the interest we have to pay on the capital which is called in at death; and the higher the rate of interest and the more regularly it is paid, the further the date of redemption is postponed.

So wrote Arthur Schopenhauer, comparing life to finance in a universe that must keep its books balanced. At birth you receive a loan, consciousness and light borrowed from the void, leaving a hole in the emptiness. The hole will grow bigger each day. Nightly, by yielding temporarily to the darkness of sleep, you restore some of the emptiness and keep the hole from growing limitlessly. In the end you must pay back the principal, complete the void, and return the life originally lent you.

By focusing on the common periodic nature of sleep and interest payments, Schopenhauer extends the metaphor of borrowing to life itself. Life and consciousness are the principal, death is the final repayment, and sleep is *la petite mort*, the periodic little death that renews.[1] Life is a temporary *non*blackness.

Schopenhauer's metaphor is striking, but less obvious metaphors are everywhere. Most of the words we use to describe our feelings are metaphors. To say you are "elated" is to say you feel *as though* you have been lifted to a high place. "Feeling high" is an out-of-control version of elation. But why is there something good about being elevated? Because in the Earth's gravitational field[2] all nonfloating animals recognize the *physical* struggle necessary to rise, and when you rise you can see the world spread out beneath you. Being elated is feeling as though you have overcome gravity. Conversely, when we feel depressed we feel as though we have been pushed down to a low place. Things are looking up, we say, or looking brighter, or less dark. These are metaphors too, rooted in our physical senses. Some metaphors are nested, traveling through several layers to their base. When we say the economy is depressed we are comparing the economy's *spirits* (another metaphor) to those of a person who feels *as though* he or she were pulled down by gravity.

Language is a tower of metaphors, each "higher" one resting on "lower" ones that preceded it. Not every word can be a metaphor; you cannot sensibly define every word in terms of other words, or else language would be meaningless. At the base of the tower are words like *push* and *down*, two of the nonmetaphorical words and concepts on which the tower rests. *Push* and *down* are understood with our bodies, because we are wetware, an amalgam of chemicals rather than silicon chips and computer code, and we experience the world through the sensations that chemicals are capable of. You cannot have lived without knowing what it is to have struggled against gravity or felt the insecurity of darkness. That is how we know that *down* and *dark* are bad and *up* and *light* are good.

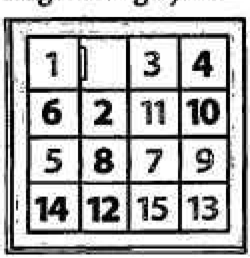
Had life arisen[3] in outer space, free of gravity and light, there would be no perceptible up or down, and hence no depression or elation. You could be disheartened, perhaps, but not depressed. You could feel full or empty but not light or heavy, bright or dark. And you couldn't take a dim view of your surroundings.

The Discovery of the Positron: Metaphors Become Real

Just as life can be viewed as a hole in the sea of darkness, so, almost a century later, Paul Dirac showed that the positron is a hole in another invisible sea. The Dirac equation, proposed in 1928, was intended to describe the essential nature of electrons in a manner consistent with both Einstein's Special Theory of Relativity of space and time and Erwin Schrödinger's nonrelativistic wave mechanics of matter, the two then recently discovered theories that together described[4] the nature of matter, space, and time. Electrons, the tiny particles that orbit the nucleus of atoms, have negative electric charge and are responsible for all the chemical properties of matter. Dirac's equation represented electrons as fast-moving relativistic objects (thereby getting their space-time properties correct) described by a probability wave (thus matching the quantum nature of their matter). Once he solved his equation, out fell mathematical solutions that miraculously accounted for the previously unexplained fact that electrons had been observed to spin about their own axes. Dirac's equation also explained various small but significant subtleties in the spectrum of light radiated by an excited electron in a hydrogen atom as it emits a quantum of light and drops to a lower energy state.

But that's not all! as TV salespeople say. Also emerging from Dirac's equation were solutions that corresponded to electrons that had *negative* energy. Negative energy is what Wall Street would call a deal breaker, because it implies that the world we know is unstable: if an electron is permitted to have negative energy, then any ordinary electron with positive energy is in an excited state relative to one with negative energy, and can therefore emit a quantum of light as it drops down into a negative-energy state. All electrons in the world would therefore cascade downward into states with arbitrarily large negative energy and radiate their way out of visible existence. But the world isn't unstable, so something is amiss here.

When computer programmers are confronted with a misbehaving program, they like to argue, tongue in cheek, that it's not an unintended bug but rather a *feature*. To circumvent the instability of his theory, Dirac came up with a bit of jujitsu, an ingenious argument that turned his bug into a feature, his weakness into a strength. He assumed that the void we live in, what physicists call *the vacuum*, is not empty, but is instead filled to the brim with negative-energy electrons, an infinite number of them at all possible negative energies between zero and minus infinity.[5] This jam-packed void, the background against which we live and act, is the metaphorical *Dirac sea*. It's the vacuum, but it's not really empty. It's full of invisible negative-energy electrons, waiting, he realized, to emerge and manifest themselves as soon as someone or something gives them a large enough jolt.



That jolt is a bolt of light. When a photon with enough momentum hits a negative-energy, negatively charged electron in the Dirac sea, it can impart sufficient energy to it so that the struck electron will pop out above the surface of the sea and become visible as a normal electron with positive energy. Having emerged, it leaves behind a hole in the sea, like the hole made by an empty square in a magic number puzzle. As you move the numbered squares around, it is the hole, the absent square, that seems to do the moving. Similarly this *absence* of an electron, this hole, moves around in the sea, and it is the hole itself of which we are aware. Just as an empty square behaves like a square, so this absence of a negatively charged electron behaves almost exactly like an electron, except that, by virtue of its absence, it appears to have *positive* charge. It is an *antiparticle*.

Renowned physicists at the time were highly skeptical of Dirac's sea. Then, in 1932, Carl Anderson at CalTech discovered a positively charged particle in cosmic rays that, except for the sign of its charge, behaved exactly like an electron. He wasn't looking for it, but it was Dirac's antiparticle, the *positron*, the jewel in the theoretical crown and the hole in the sea. Dirac received the 1933 Nobel Prize in Physics, sharing it with Schrödinger, and Anderson received his in 1936, sharing it with Victor Hess, who discovered the cosmic radiation that became the source of many soon-to-be-discovered additional particles.

Absence Is a Presence

Schopenhauer's notion of eternal sleep as normality and life as brief temporary periods of punctuated antisleep corresponds to Dirac's picture of the positron as a brief fluctuation in the vacuum. Schopenhauer saw the bad things in life—sickness and pain—as positive, in the sense that they are primary. He saw the good things—health and pleasure—as the mere secondary absence of the bad:

> Just as we are conscious not of the healthiness of our whole body but only of the little place where the shoe pinches, so we think not of the

totality of our successful activities but of some insignificant trifle or other which continues to vex us. On this fact is founded what I have often before drawn attention to: the negativity of well-being and happiness, in antithesis to the positivity of pain.

I therefore know of no greater absurdity than that absurdity which characterizes almost all metaphysical systems: that of explaining evil as something negative. For evil is precisely that which is positive, that which makes itself palpable; and good, on the other hand, i.e. all happiness and all gratification, is that which is negative, the mere abolition of a desire and extinction of a pain.

This is also consistent with the fact that as a rule we find pleasure much less pleasurable, pain much more painful than we expected.

A quick test of the assertion that enjoyment outweighs pain in this world, or that they are at any rate balanced, would be to compare the feelings of an animal engaged in eating another with those of the animal being eaten.

Good as the absence of evil suggests a certain boredom that may not be tolerable for long.

Is good the opposite of evil, the absence of evil, or simply independent of evil? Schopenhauer's perception of the primary status of the negative was reflected again a century and a half later by the Israeli poet Yehuda Amichai:

The Precision of Pain and the Blurriness of Joy

The precision of pain and the blurriness of joy. I'm thinking
how precise people are when they describe their pain in a doctor's
office. Even those who haven't learned to read and write are precise:
This one's a throbbing pain, and this one's
a wrenching pain, and this one gnaws, this one burns and
this is a sharp pain and this

is a dull one. Right here. Precisely here, yes, yes.
Joy blurs everything. I've heard people say
after nights of love and feasting, It was great,
I was in seventh heaven. And even the space man who floated
in outer space, tethered to a space ship, could only say, Great,
wonderful, I have no words.
The blurriness of joy and the precision of pain—
I want to describe with a sharp pain's precision
happiness and blurry joy. I learned to speak among the pains.[6]

Leszek Kolakowski, the Polish philosopher and historian who lived through Stalinism and died in 2009, also regarded evil as a positive quality:

> The Devil is part of our experience. Our generation has seen enough of it for the message to be taken extremely seriously. Evil, I contend, is not contingent, it is not the absence, or deformation, or the subversion of virtue (or whatever else we may think of as its opposite), but a stubborn and unredeemable fact.

G. K. Chesterton experienced goodness as more than the mere absence of badness. In his essay "A Piece of Chalk" he wrote:

> Virtue is not the absence of vices or the avoidance of moral dangers; virtue is a vivid and separate thing, like pain or a particular smell. Mercy does not mean not being cruel, or sparing people revenge or punishment; it means a plain and positive thing like the sun, which one has either seen or not seen.
>
> Chastity does not mean abstention from sexual wrong; it means something flaming, like Joan of Arc. In a word, God paints in many colours; but He never paints so gorgeously, I had almost said so gaudily, as when He paints in white.

Goethe, who conducted his own experiments on the perception of color, noticed that when white light is split by a prism or a diffraction grating, the colors of the rainbow arise at the boundaries between light and darkness. Just as electric charges can be positive and negative, and as magnetic poles can be north or south, so, according to Goethe, darkness is the polar opposite of light rather than its absence, and colors arise from the interaction between the poles.

As I recounted in chapter 1, the apartheid government saw white as the positive quality and blackness as the lack of it. In modern disagreement, a Broadway musical whose billboard I walked by a few days ago near Times Square advertises *Spider-Man, Turn Off the Dark*. Which of these views is correct? All, probably. Metaphors are analogies, focused on one quality of a phenomenon but not the entire phenomenon itself. Hence in Schopenhauer's analysis of sleep, it is sleep's periodicity that resembles the coupons of a bond. Like adages, metaphors capture only partial truths, not entireties. As schoolboys in love we used to revel in the conflicting adages "Absence makes the heart grow fonder" and "Out of sight, out of mind," recognizing the partial truth of both of them.

Spinoza, as we will see one chapter hence, is more evenhanded. His theory of emotions regards both *pleasure* and *pain* as independent qualities of human experience, neither one being either the reflection or the absence of the other. If Spinoza is correct, it must be possible to experience both pleasure and pain simultaneously rather than as opposites. I think I can.

Abandoned lovers and lapsed believers can testify only too well that absence is indeed a presence.

The Positron as Metaphor, Fact, and Theory

Dirac began with an equation, simple and elegant: $-i\hbar\gamma^{\mu}\partial_{\mu}\psi + mc\psi = 0$

Making it work correctly required its interpretation as a metaphor: the sea. This combination, theory plus metaphor, successfully predicted the existence of a particle no one had seen before. A metaphor grounded in a theory can have more power than either alone.

Dirac found the positron to be a hole in the sea of electrons. He

could, had he started with positrons, have found the electron to be a hole in a sea of positrons. Either view works. The key notion is that of symmetry, the absence of one requiring the presence of the other.

The later development of quantum field theory, also pioneered by Dirac, treated electrons and positrons more evenhandedly and less picturesquely, describing both of them as the oscillations of a quantum field that extend throughout space and time, and led to the same results as those of the Dirac sea, but with less need for imaginative effort. Theories discovered by great leaps of individual insight eventually become transformed into formulas anyone can learn.

ANALYTIC CONTINUATION

Schopenhauer viewed sleep as the metaphorical interest on a loan because of their similar regularities. Taking an analogy based on matching regularities and then extending it into distant regions is a time-honored trick of mathematicians. It's called *analytic continuation.*[7]

Physicists love to extend their theories too, and to impose their extended definitions on us without drawing attention to the subtle transformation. We measure household distances with a ruler or tape measure, but how do you measure the distance to faraway galaxies that the science sections of newspapers so merrily quote as being 100 million light years away? You can't lay out measuring sticks across the universe.

The distance to a galaxy is also an unspoken kind of analytic continuation. One of the ways galactic distances are measured is by observing Cepheid variables, stars whose visible brightness varies. Their true luminosities ("luminosity" is the technical term for their light output, or brightness) have been found to pulsate in a predictably regular way, so that the frequency of their pulsation depends on their luminosity. By measuring the frequency, you can tell something about the true luminosity of these stars. I say "true" luminosities because I want to distinguish between true and apparent luminosities. The true luminosity is the actual light emitted by the star; the apparent luminosity is how bright the star looks, as determined by the light that enters your eye. The farther away a star is, the less light from it reaches your eye. Because the light from a star a distance R away radiates out over a sphere of surface $4\pi R^2$, the apparent luminosity decreases with distance inversely proportional to R^2. When you look at a Cepheid variable in a distant galaxy through a telescope, you see its apparent luminosity, but the frequency of the pulsation tells you its absolute luminosity. From the ratio of the true and apparent luminosities you can calculate the distance R to the star.[8]

What an indirect way this is of measuring something as apparently simple and intuitive as distance! The distance to a galaxy has been determined by making use of a regularity of these weird stars that links the quantity of light emitted to the frequency of their pulsation, a "law" that is believable because it can be explained by plausible models of stellar evolution. This measurement of distance makes use of advanced physics rather than Pythagorean geometry. Intergalactic light-years, the circumference of the Earth, the gap between my head and the screen on my laptop, and the separation between atoms—each of these distances is "measured" rather than observed by different methods. Most of these measurements involve the analytic continuation of the notion of distance through the use of models and theories.

I like this observation:

> The ultimate goal would be: to grasp that everything in the realm of fact is already theory.
>
> —Goethe, *Maxims and Reflections*

DIG WE MUST

Why models? Because the inanimate world is filled with quasi-regularities that hint at deeper causes. We need models to explain what we see and to predict what will occur. We use models for envisioning the future and influencing it.

The world of people is unpredictable and begs for divination as well. At every moment we face choices with uncertain outcomes. Each decision, even one made on the spur of the moment, involves, just beneath the surface, some imagined model for how the future may evolve and how our choices will affect it. We are always weighing the odds, estimating the relative importance of causality and chance. Without time, there is no need for action.

As time passes, possibilities narrow. Because our lifetime is finite, time, choice, risk, and reward are of the essence. Unless you can live in the perpetual present, you need theories and models to exert some control. Theories and models are a kind of magic, and the builders of successful ones, like Dirac, are shamans bridging the visible and invisible worlds.

A MODEL AIRPLANE: THE ZIPPY

My earliest recollection of models is of the scaled-down airplanes we used to build from model kits in grade school. When I was eight my mother let me take the bus on my own down to Jack Lemkus in St. George's Street in Cape Town and choose a kit to take home. Some kits were too difficult and time-consuming for an eight-year-old's patience and skills, requiring days of careful assembly; others, the simple gliders, were too unsatisfyingly easy to piece together, taking only a few minutes. One had to find a level of challenge that was difficult and yet surmountable.

The only plane I built successfully was a Zippy. The kit contained long thin strips of lightweight balsa wood used to create the frame of the plane. It also included flat sheets of the same wood with preprinted cross-sectional inserts that prevented the frame from collapsing. (Tropical balsa is so strong, light, and flexible that the De Havilland Mosquito, a genuine full-size World War II British combat aircraft, was partially constructed of this wood.) A block of balsa had to be carved and sanded and then glued into the nose to hold the propeller. You pinned the plan to your mother's bread-kneading board and used dressmakers' pins to force the long balsa strips to curve along the preprinted arcs that defined the struts. Then you cemented them to each other with airplane glue. When the glue dried, you removed the pins and relied on the cement to maintain the curvature of the stressed beams. Then you glued the sides of the frame to the cross-sectional inserts.

The fuselage was translucent tissue paper cemented to the balsa frame, trimmed, then dampened with water to shrink it taut. When it was dry, you lacquered and painted it to make it stiff and realistic. The engine was merely a long rubber band that ran the internal length of the fuselage, from the propeller block at the nose to a hooked pin inserted into the tail. You rotated the propeller many times to wind up the rubber band and then let it loose. The propeller accelerated and spun as the band unwound, and the plane, it you were lucky, took a brief flight of perhaps ten seconds at best. If you were really ambitious about airplane models—I wasn't, though I admired such ambition in some of my friends—you followed every instruction very carefully, especially sanding off any excess glue on the frame before overlaying the tissue so as to leave no imperfections at all.

I assume that somewhere in the universe of actual airplanes there was or had been a Zippy. My model Zippy was smaller and lighter than the putative actual Zippy; it lacked seats, ailerons, and functioning windows and doors; it was made of totally different materials. Why did they call it a model?

TYPES OF MODELS

I'm very well acquainted, too, with matters mathematical,
I understand equations, both the simple and quadratical,
About binomial theorem I'm teeming with a lot o' news
With many cheerful facts about the square of the hypotenuse.

I'm very good at integral and differential calculus;
I know the scientific names of beings animalculous;
In short, in matters vegetable, animal, and mineral,
I am the very model of a modern Major-General.

So sang Gilbert and Sullivan's Major General Stanley in *The Pirates of Penzance* of 1879. The mathematical expertise he sings of is remarkably thorough, not only for 130 years ago but for contemporary financial modelers too. Stanley claims to be what we now call a role model or exemplar, a particular specimen that exemplifies the ideal qualities of a class. That's one use of the word *model*. Model airplanes are another. We also refer to the Model T, fashion models, artists' models, a weather model, an economic model, the Black-Scholes Model, the Standard Model. What do we mean when we call something a model?

The Model T

The Model T is a type of Ford, one of a class of things belonging to the Ford category. The Model T is an instance, not everything a Ford can be.

Fashion Models

A fashion model displays clothing or cosmetics. What's important about a fashion model is the exterior: looks, physique, aura. The rest is more or less irrelevant, except insofar as auras and exteriors reflect interior qualities. My daughter was once a hand model in a web advertisement. When you're a model, only parts of you are important. A person is the real thing.[9]

Artists' Models

An artist's model is a proxy for the real thing. A mannequin is a proxy for a proxy, two degrees of separation. The work of art that uses the proxy is its own real thing, complete in its own way and no longer a proxy at all.

A Weather Model

A computer model of the weather tries to predict the future weather from the weather today. "Weather" is an abstraction for a collection of an indefinite number of qualities and quantities and the way they vary over the short term, among them temperature, pressure, humidity, and wind speed. A weather model specifies the relevant variables and links them through a set of dynamical equations from physics and chemistry that represent the effects of sunshine, clouds, heat, moisture, evaporation, and air and water currents as they propagate through the atmosphere and along the surface of the Earth as it rotates about its axis and about the sun.

A weather model is much more clearly *not* the weather than the Zippy model is not the airplane. The Zippy is instantly recognizable as a representation of the airplane. The weather model is recognizable as a model of the weather only for someone with the right education.

A weather model's equations are a limited and partial representation of a limitlessly complex system. One cannot model the physics, chemistry, and biology of all the chemicals in the atmosphere and their effect on every species on Earth. There is always the danger that one has omitted something ostensibly negligible whose tail effects over long times are crucially important. This is what makes the predictions of global warming the subject of legitimate debate.

Economic Models

An economic model aims to do for the economy what the weather model does for the weather. It too embodies a set of equations to represent the interactions of people and financial institutions. But an economy is an even more abstract concept than the weather. Supply, demand, and investors' utility, to name just a few of many possible variables in the model, are much harder to define (let alone quantify) than temperature and pressure. When you model "the economy" and "the market" you are modeling high-level abstractions.

Friedrich Hayek, the Austrian economist who received the 1974 Nobel Memorial Prize in Economics, pointed out that in the physical sciences we know the macroscopic through concrete experience and proceed to the microscopic by abstraction. For example, the first theories of gases dealt with volume, pressure, temperature, and heat, all directly accessible to our senses. Centuries later we understand pressure as the kinetic energy of invisible microscopic atoms. The atoms, though we consider them real, are more abstract than the pressure and temperature that we perceive directly. In economics, Hayek argued, the order of abstraction should be reversed: we know the individual agents and players from concrete personal experience, and the macroscopic "economy" is the abstraction. If the correct way to proceed is from concrete to abstract, he argued, in economics we should begin with agents and proceed to economies and markets rather than vice versa.

The difficulties one encounters in modeling economic abstractions are illustrated by attempts to deal with the notion of market liquidity. Liquidity is the metaphorical quality that makes trading possible; it connotes the easy availability of counterparties to buy something you want to sell or sell something you want to buy, and its disappearance in states of fear causes the great damage that characterized the recent global financial crisis. Everyone thinks he knows what liquidity means, yet no one has yet adequately defined and quantified it.

The Black-Scholes Model

Black-Scholes, as it's commonly referred to by financial practitioners, is the most celebrated and widely used model in all of economics. I spent 17 years of my professional life at Goldman Sachs & Co. extending the Black-Scholes option pricing model in a variety of directions.

A stock option is a kind of lottery ticket you can buy whose future payoff depends on the future moves of the stock price, up or down. It provides reward (if you guess the direction of the move correctly) in exchange for risk (the chance that you guess wrong and lose the price of the ticket). The Black-Scholes Model tells you how to estimate the value of an option in terms of the stock price's risk.

Risk versus reward is the overwhelming issue in finance: how much potential future reward does it take to justify the risk of losing your money when you make an investment? Risk connotes the *possibility* of harm, and so financial theory is intimately bound up with the mathematical theory of probability, which originated centuries ago in connection with the attempt to estimate gambling odds. Buying a stock is a symmetrically risky endeavor: if its market price goes up after purchase, you make money; if the price goes down, you are proportionately harmed. A call option is an investment in only the upside of the stock. If the stock price has risen by some amount at expiration, the option will have made you that many dollars, but if the stock price has dropped, you receive no payoff and lose only the price you paid for the option. The Black-Scholes Model tells you what the value of the option is.

An option is a complex conceptual machine. Its value rises when the stock price rises and falls when the stock price falls. Black-Scholes provides a recipe for *manufacturing* a call by borrowing money to buy shares of the stock. The model tells you exactly how many shares to buy initially and then, at every future instant of time and at every future stock price, how much additional stock to buy or sell so that the stock you own will replicate the payoff of the option contract. The value of the option is the total cost of its manufacture, the cost of all the required trading with borrowed money. The Black-Scholes formula explains how the option value—the estimated cost of trading—depends on the stock price, the interest charged for borrowing, and the riskiness of the stock itself.

Just as a weather model makes assumptions about how fluids flow and how heat undergoes convection, just as a soufflé recipe makes assumptions about what happens when you whip egg whites, so the Black-Scholes Model makes assumptions about the riskiness of stock prices, that is, about how stock prices fluctuate. Black-Scholes assumes that stock prices move smoothly but randomly with a definite volatility, a fixed degree of fluctuation. Given the assumptions, you can figure out the net cost of manufacture. That cost is the fair price of the option, *assuming the validity of the model*. Just as my father could figure out what to charge for homemade batteries by estimating the cost of lead, casting, labor, sulfuric acid, and Bakelite, just as a dessert chef can figure out how much to charge for a soufflé based on ingredients, labor, and waste, so Black and Scholes could estimate how much it would cost to manufacture an option.

But there is a crucial difference between the assumptions made by the Black-Scholes Model and the assumptions made by a soufflé recipe. Our knowledge about the behavior of stock markets is much sparser than our knowledge about how egg whites turn fluffy. Fluids and egg protein don't care what people think about them; markets and stock prices do. Like a weather model (but even more so), Black-Scholes is an ingeniously clever mental model of a complex system, an elegant mechanism that, in trying to reflect the actual world in a short description, must reduce its intricacy. That reduction makes the model usable but simultaneously limits its usefulness.

The Standard Model

The Standard Model, for which Sheldon Glashow, Abdus Salam, and Steven Weinberg received the 1979 Nobel Prize in Physics, is a unified description of quarks and leptons, the smallest elementary particles, and the forces between them. The description incorporates into one coherent framework James Clerk Maxwell's nineteenth-century theory of electromagnetism, the 1928 Dirac theory of the electron, and Enrico Fermi's 1934 theory of radioactive beta decay, a framework in which all of these apparently disparate forces are merely superficially different aspects of a single, more general force. I spent the first part of my professional life as a theoretical physicist, working on tests of the Standard Model.

The Standard Model is not really a model at all; it is a description, and hence a *theory*. A theory attempts to provide an accurate portrayal of the nature of things, unifying the outward with the inward, not just saving the appearances but identifying their essence. I say "attempts" because a theory can be right or wrong. What makes something a theory is the way it tries to depict and explain. When someone proposes a model, you can ask "Why?" and expect arguments that make the analogy plausible. When someone proposes a theory, "Why?" is less important. A model is the construction of an analogy. A theory is the linking of the outer with the inner.

The process of unifying several previously disparate theories is a bit like confirming the existence of a never-observed bird from a small fragment of its birdsong. From the song fragment you deduce a morphology; from the morphology you predict the entire song. To confirm the existence of the bird you must then find more fragments of the same bird's song, as predicted. If you hear them, you confirm the theory. The bird itself is never seen.

From fragmentary evidence Glashow, Weinberg, and Salam figured out the entire song; one of its predicted disharmonies was small amounts of *parity violation*, the technical name for a phenomenon in which more particles move to the left than to the right. While *up* and *down* are absolute directions in a gravitational force field, directly perceptible by the human body, until the mid-1950s *left* and *right* had seemed to be conventions of speech rather than physical realities. My left is your right, but our ups and downs are the same. Then in the 1950s physicists discovered that radioactive beta decay, a force whose consequences are also perceptible by the body, does distinguish *left* preferentially from *right*. One can absolutely define *left* and *right* by the direction of the asymmetry in the distribution of particles produced in beta decay. That's a fact.

The Standard Model predicted additional, previously unobserved left-right asymmetries in nature. My PhD thesis of 1973 proposed an experiment to detect these asymmetries in high-energy electron-proton scattering, an experiment in which one smashes spinning electrons into stationary protons and then observes the distribution of the electrons as they bounce off the target. I calculated the size of the predicted asymmetry in the Standard Model. An asymmetry of the appropriate size was finally observed, as predicted, at the Stanford Linear Accelerator Center in 1978. The experiment provided the final stamp of approval and converted the standard model into the Standard Model. The results were welcomed as "the long elegiac salute given to the end of an age."[10] The elegy was the full melody of the Standard Model. The age was the period of pell-mell discovery of new subatomic particles, from the electron in 1898 through neutrons, pi mesons, and their siblings, culminating in the discovery of the quarks inside them and the W- and Z-bosons that mediated their interactions. We are now more than 30 years into the age beyond that. Though physicists can invent many new orchestras consistent with the fragmentary music of gravity and cosmology, none of their instruments has yet been discovered.

THE NATURE OF MODELS

There Is Always a Gap

My Zippy wasn't the actual airplane itself, though it bore some similarity to the plane. Similarity lies in the eyes of the beholder and creator.[11] My model Zippy was created with the intention of reproducing some small number of important features on a smaller scale. My Zippy looked like an airplane. Its construction—frame, struts, and fabric to create a light yet strong structure—was sound from an engineering point of view and similar in style (though not in size and material) to the real Zippy. And it could (briefly) fly.

The realistic appearance, the structure beneath the skin, and the ability to fly made the Zippy a suitable model for me at age eight. The structure was important, though reproducing it was hard work. At age three or four I would have been happy with a rudimentary wooden airplane that I could have zoomed through the air with my hand while making throaty airplane noises. If I had been a few years older, I would have wanted a combustion engine and radio control. Had I been an aircraft designer, the ability to test the aerodynamic lift and stability would have been critically important. But, however complex, all of these models are limited when compared with the real thing. There is a gap between the model and the object of its focus. The model is not the object, though we may wish it were.

A dreamed-of counterexample is the model created by Pygmalion: a statue of a woman so beautiful that he fell in love with it. This is a not uncommon occurrence in the worlds of finance and nutrition, both of which abound with experts reluctant to abandon their models in the face of evidence of their unreality. Pygmalion was lucky; Aphrodite granted his request to bring the statue to life, he called her Galatea, and they lived happily ever after.

I. MODELS

An Analogy, a Caricature, a Fetish

A model is a metaphor of limited applicability, not the thing itself. Calling a computer an electronic brain once cast light on the function of computers; nevertheless a computer is not an electronic brain. Calling the brain a computer is a model too. In tackling the mysterious world via models we do our best to explain the thus far incomprehensible by describing it in terms of the things we already partially comprehend. Models, like metaphors, take the properties of something rich and project them onto something strange.

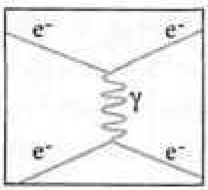
A good example is the collective model in nuclear physics, for which Aage Bohr (the son of Niels Bohr), Ben Mottelson, and James Rainwater received the Nobel Prize in Physics in 1975. The collective model regards the core of the nucleus as a drop of dense incompressible fluid that interacts with a small number of so-called valence protons and neutrons outside the core. Of course, the core itself really[12] consists of protons and neutrons held together very tightly by their mutual attraction, but if you think of it as a liquid drop that, when excited, can oscillate, vibrate, and rotate, then you can figure out the energy of its collective excitations and their interaction with the protons and neutrons outside the core. With this model that combined a fluid core with an external shell, Bohr, Mottelson, and Rainwater were able to explain the excited states of uranium and other heavy nuclei

The picture of the nucleus as a drop of water is a limited analogy. Regarding the nucleus as a liquid drop is very different from describing the electron using the Dirac equation. The Dirac equation, even if it eventually turns out to be not quite the absolute truth, will still have been an attempt to intuit the essential nature of the electron. The collective model merely compares the nucleus to a drop of water.

A model is a caricature that overemphasizes some features at the expense of others. It focuses on parts rather than the whole. It is a fetish in which the importance of one key part of the object of interest is obsessively exaggerated until it comes to represent the object's quintessence, such as a shoe or corset standing in for a woman. (Is that perhaps why most modelers are male?) But the shoe or corset isn't the woman; it is just the most important part of the woman for this model user. Once you understand that a model isn't the thing but rather an exaggeration of one aspect of the thing, you will be less surprised at its limitations.

Let Someone Else's Fingers Do the Walking

Thinking for yourself is hard work, and models save mental labor. Like the vacuum cleaner and washing machine that promised to liberate suburban housewives of the 1950s from drudgery, models provide easy and automated ways of letting other people do the thinking for you.



When I worked on my PhD thesis to test the Weinberg-Salam Model in the early 1970s, I carried out each calculation using Feynman diagrams, the cartoonlike representations invented by Richard Feynman in the late 1940s to systematize and enumerate the ways particles interact during collisions. Using a formal set of rules that Feynman developed with his inimitable blend of mathematics and intuition, rules that were later justified by the more rigorous mathematics of Freeman Dyson,[13] I elaborated and drew all the possible diagrams that could occur in the Standard Model, and then, using Feynman's rules, translated each picture into a mathematical formula and evaluated it. The calculations were carried out with pen and paper and took hundreds of pages. To check the accuracy I repeated each calculation at least twice, the second time without looking at the first.

Feynman's diagrams and rules are bookkeeping by picture, a Tinkertoy algorithm that miraculously captures all the details of quantum mechanical forces in the Standard Model via a series of stick-and-vertex diagrams; they allow people less talented than Feynman to use his pictures to perform the most complex calculations carefully and correctly. Like all great advances in physics, they codify and make routine what was formerly almost impossible to think about. Only Feynman could have invented it; now wet-behind-the-ears graduate students can churn out page after page of accurate calculations. Indeed, by the time I stopped doing physics in 1980 I knew two professors in Wisconsin who had programmed computers to generate the diagrams in the Standard Model, translate them into formulas, evaluate them, and mechanically calculate the magnitude of the effects predicted. All that remained was to write the paper.

Einstein similarly made calculations easier for lesser physicists when he discovered the *theory* of special relativity. Hendrik Lorentz, for whom the Lorentz-Fitzgerald relativistic contraction of rapidly moving rods was named, had more or less come to the same conclusions as Einstein. But he did so by carrying out difficult and complex calculations using elaborate *models* of the atoms within rods, determining what happened to the forces between them as they moved. Einstein replaced these struggles with simple but deep analyses of what it means to talk about the length of a rod. Now high school students can calculate the changing size of objects and the changing periods of clocks as they move. Lorentz's model needed justification. Einstein's theory is exact fact.

In both physics and finance the first major struggle is to gain some intuition about how to proceed; the second struggle is to transform that intuition into something more formulaic, a set of rules anyone can follow, rules that no longer require the original insight itself. One person's breakthrough thus becomes everybody's possession.

A Model Is a Little Language

It takes hard work to master a model. In *Zen in the Art of Archery*, Eugen Herrigel describes his repetitive, initially futile struggles with the Japanese bow and arrow, until finally he was able to transcend the battle for conscious competence and achieve unconscious skill, pulling back the string and launching the arrow mindlessly, carelessly, and accurately at the target. Anyone who plays tennis only occasionally will have noticed that, after a hiatus, one often does better the first time on the court than the second. As one tries to try to make improvements, one gets worse. As Spinoza wrote, "The body can by the sole laws of its nature do many things which the mind wonders at."

The most valuable knowledge is that which has become unconscious and intuitive. Focusing one's eyes, grasping, manipulating, chewing, crawling, walking, or speaking—one begins by struggling to do these things and ends by doing them without thinking or struggling. Practice to the point of automaton-like competence is necessary. Until you can do something without thinking, you can't progress further up the hierarchy of linguistic or modeling metaphors. If you eschew the help of the mental machines or models created by your intellectual forebears, you have to think through everything for yourself, every time. There are occasions when the capacity to think from scratch is important, but most of the time it's best to take your foundation for granted.

So many of our acquired abilities move from conscious struggle to unconscious achievement that some writers have theorized, implausibly but fascinatingly, that every unconscious human ability (even digestion, to take an extreme example) was first learned by conscious efforts of the will, and that it is the failure to achieve unconscious automation that leads to various kinds of mental ailments.[14] Suffice it to say that when you have digested a model or language well—and a model *is*, like language, a framework for communication—then, with it inside you, you gain power.

Models Reduce the Number of Dimensions

The world is impossible to grasp in its entirety. We can focus on only a small part of its vast confusion. Models project multidimensional reality onto smaller, more manageable spaces where regularities appear and then, in that smaller space, allow us to extrapolate and interpolate from the observed to the unknown. At some point, of course, the extrapolation will break down. What's amazing is how well this strategy of reduction can work, especially in the physical sciences.

Models in finance use the same strategy. Companies that issue stock are multidimensional. You can evaluate them with respect to many metrics: management, earnings, debt, credit quality, patents generated, and so forth. The stock market's job is to collapse all of these qualities and quantities into one number, the stock price, measured on a one-dimensional scale of dollars. But price alone doesn't indicate relative value: Who knows which is a better deal, apples at $2 a pound or oranges at $3? IBM at $100 per share or Microsoft at $250? Financial models attempt to answer these questions. To do so they project the company onto an axis that measures value more usefully than dollars do. Though price is a fact, value depends on the observer. For office space, value might be price per square foot. For fruit it could be taste, or the quantity of vitamins and fiber, depending on who the model serves. For stocks, value can be measured by the ratio of price to earnings; for bonds, by the yield to maturity. All of these axes represent a view on the source of value, and hence a model.

The Dangers of Extrapolation

Models project a detailed and complex world onto a smaller subspace. But extrapolation in the smaller space can be unreliable. The supposed benefits of estrogen supplements for postmenopausal women and the advantages of margarine over butter for preventing arterial plaque have turned out to be dubious. Estrogen can slow osteoporosis, but it's associated with an increase in various kinds of cancer. Cholesterol intake from food isn't directly equivalent to cholesterol output into plaque, though it is plausible that they are linked. Eggs and butter contain many substances that are good for you, whereas margarine contains many that aren't. Models are simplifications, and simplification can be dangerous.

THE NATURE OF THEORIES

A weather model's equations are a model, but the Dirac equation is a theory. What *is* a theory, and why do I call the Dirac equation one? The Online Etymological Dictionary lists a definition of the word *theory* dating from 1638 as "an explanation based on observation and reasoning." Wikipedia cites Francis Cornford, an English scholar, suggesting that the practitioners of the Greek religion Orphism used the word *theory* to mean "passionate sympathetic contemplation." Both phrases are well chosen.

Models are analogies; they always describe one thing relative to something else. Models need a defense or an explanation. Theories, in contrast, are the real thing. They need confirmation rather than explanation. A theory describes an essence. A successful theory can become a fact.

> The ultimate goal would be: to grasp that everything in the realm of fact is already theory. . . . Let us not seek for something behind the phenomena—they themselves are the theory.
>
> —Goethe, *Maxims and Reflections*

In *Science and Reflection*, Martin Heidegger comments on the contemplative nature of theoretical insight:

> The word "theory" stems from the Greek verb *theorein*. The noun belonging to it is *theoria*. Peculiar to these words is a lofty and mysterious meaning. The verb *theorein* grew out of the coalescing of two root words, *thea* and *horao*. *Thea* (cf. theater) is the outward look, the aspect, in which something shows itself. . . . The second root word in *theorein*, *horao*, means: to look at something attentively, to look it over, to view it closely. Thus it follows that *theorein* is *thean horan*, to look attentively on the outward appearance wherein what presences becomes visible and, through such sight—seeing—to linger with it.

The related Greek word *aletheia* means "the state of not being hidden," and suggests that theater (*thea*) hides, and that the role of theory is to make evident what is hidden. That's the way I have felt whenever I've done research, in physics or finance. The creator of a theory is attempting to discover the invisible principles that hide behind the appearances. Evolution is a theory; so are the theory of dreams, Newton's laws of motion, and the Standard Model. A theory is (potentially) deep; a model, even when efficacious, is shallower. Theories describe absolutes, like Moses descending from the mountain with the Ten Commandments, or like God commanding, "Let there be light!"

You can see why intuition, the union between object and subject, bears such a close relationship to theories. A theory doesn't simplify. It observes the world and tries to describe the principles by which the world operates. A theory can be right or wrong, but it is characterized by its intent: the discovery of essence. A theory is an absolute nonmetaphorical insight, and this is why the abstractions of mathematics are often more suitable than words for formulating theories.[15] When I wrote papers in physics, I dreamed of discovering a theory that would be true and transcendental and would survive my lifetime. Newton, Lagrange, Hamilton, Darwin, Maxwell, Freud, Einstein, Bohr, Schrödinger, and Gell-Mann, to name just a few, accomplished this. Therefore it's possible.

Newton's Universal Law of Gravitation is a theory. It postulates that the force between any two masses (the moon and the Earth, the Earth and an apple) is proportional to the product of their masses and inversely proportional to the square of the distance between their centers. Newton's Second Law of Motion, a theory too, dictates that the gravitational force accelerates each particle inversely proportional to its mass. Solving these equations, Newton deduced that planets must traverse elliptical paths around the sun, thereby justifying Johannes Kepler's empirical laws of planetary motion.

Newton's theory is general and precise. The gravitational force is inversely proportional to *exactly* the square of the distance between the planets; Newton was confident that the power of the distance is precisely 2. Had he been a social scientist performing statistical regressions in psychology, economics, or finance, he would probably have proposed a power of 2.05 ± 0.31.

A theory is not a fetish; when it is successful (see the quantum theory of electricity and magnetism in chapter 4) it describes the object of its focus so accurately that *the theory becomes virtually indistinguishable from the object itself.* Maxwell's equations *are* electricity and magnetism; the Dirac equation *is* the electron; the Weinberg-Salam model of weak and electromagnetic interactions matches the electrons and quarks in almost every detail, as closely as one can measure. You can layer metaphors on top of the equation, but the equation is the essence.[16]

A theory doesn't have to be complete or unmodifiable. Now, more than 106 years after Einstein's Special Theory of Relativity and 96 years after his General Theory, we know that Newton's laws are not quite accurate. But there is still something recognizably absolute about their intent, and they have not been relegated to the status of a model despite their limitations. They are theories that are not exactly right, but they are not models.

Models splinter when you look at them closely. Theories are irreducible, the foundations on which new metaphors can be built. Theories are the thing itself; when you look closely, there isn't anything more to see. The surface and the object, the outside and the inside, are one.[17]

MONOCULAR DIPLOPIA

Twenty-five years ago I accepted the offer of a free group tennis lesson while on vacation at a Caribbean resort. The coach put six of us onto a single court and made us practice volleying in pairs with our partners across the net. I was in the pair on the extreme right. The gentleman diagonally across the net from me, on the extreme left, decided to be a hero and slammed the ball at his partner as hard as he could. Instead it flew diagonally across the net and directly into my right eye. I fell to the ground and tried to figure out what was happening.

It hurt, but not too badly, and in the end I ignored it and assumed that the black floaters I saw were just what you should expect to see if you got hit in the eye. A week later, when I returned to New York, I realized that things around me were beginning to look as though I were living in an aquarium. I went to an eye doctor and, just in time, discovered that my retina had a "horseshoe" tear and was beginning to detach, like Scotch tape starting to peel from one corner. An eye surgeon resealed the retina to the back of my eye by welding it with lasers beamed through the front and with liquid nitrogen applied to the back. I still go for regular checkups to make sure my retina stays in place and have been warned to watch out for sudden changes in vision.

One morning 23 years later, in early 2008, I woke up and took a cab to work. Looking at the meter, I realized something wasn't quite in order. Used to testing my vision, I shut my left eye. Everything looked okay. Then I shut my right eye and immediately realized that all the LED digits on the taxi meter were doubled. Uh-oh, I thought. My retina again. I told the taxi to turn around and take me directly to my retinal specialist.

A careful man, the specialist tested my vision, measured the pressure inside my eye, dilated my pupils, visually examined my retina from all angles, had a technician do an ultrasound scan of my retina and the layers below it, and finally had another technician inject a fluorescent green dye into my bloodstream and then film the back of my retina to check the state of its capillaries. He saw nothing awry.

Monocular diplopia, the fancy name for seeing double in one eye, is a rare phenomenon. It cannot be a problem of accommodation or so-called squintness that occurs owing to a muscular imbalance between both eyes. My doctor sent me to an ophthalmic neurologist, who, after carefully checking that I could walk in a straight line, stand on one leg, and follow his index finger with my eyes, concluded that I had no neurological problems and told me to go away and not come back.

Nevertheless the diplopia persisted, waxing and waning inexplicably, sometimes almost disappearing for a few hours and then returning with full force. I had trouble reading. I tested my vision constantly, closing one eye and then the other while trying to read distant street signs. I used my thumb and index finger to create a small pinhole through which I peered. When I did so the signs became sharp, which told me that my retina was working well and that the trouble was therefore with the diffraction. Light from each letter in a distant sign reaches your retina along many parallel paths through your lens and the vitreous fluid, and somehow the paths weren't converging to one retinal point, hence the doubling. My retinologist could see nothing especially wrong with my retina and had no further suggestions for treatment or diagnosis, and so, perforce, I became my own general contractor.

I subcontracted parts of myself out to different specialists, trying to figure out what was happening. I read the package insert on a medicine I had been taking and saw that it listed double vision as a rare side effect. Eventually I learned that almost all drugs list double vision as a possible side effect, but when it occurs it's usually binocular diplopia. A very good optometrist I knew for years found that my astigmatism changed even during the course of a brief visit in which he tested my vision. He told me it might be due to stress. But neither of us could understand how stress could affect the curvature of my lens.

Years earlier I had had a cataract removed from my left eye; its lens was replaced by a plastic one that worked fine. But, I reasoned, a plastic disk floating inside the capsule behind the cornea might have suddenly shifted or tilted a small amount the day the double vision began, and so caused my problem. It might still be loose and moving about each day, hence the fluctuations.

I visited doctors recursively, one doctor suggesting the next. The *n*th doctor was the cataract surgeon who had originally inserted my artificial lens, and I asked him to check whether it had shifted. It hadn't. But he could theorize too: most likely, he said, there were some small, undetectable distortions in the rods and cones of my retina, deformations too small to show up on any retinal scan. The rods, which should have pointed straight up, were most likely askew, blades of plastic grass on trampled Astroturf. Nothing anyone can do about it, he said.

Once he had delivered this diagnosis, something impelled him to measure the curvature of my cornea, a test no one had done in the preceding months. A small optical device shines perfectly circular concentric rings of light onto your cornea and photographs the reflections; their deviation from perfect circles indicates the contours of the cornea. Mine was badly buckled on the nasal side. He informed me curtly and concisely that my problem was not the retina. I had keratoconus, a disease in which the cornea progressively thins and ultimately bulges under its own weight, tugged at by the Earth beneath it. (In that case, I reasoned, I should see better in outer space or in free fall, a diagnostic test I must remember to recommend when space flight becomes cheaper.) The resultant bulges in the cornea refract light improperly and cause the diplopia. I didn't like the name keratoconus, which reminded me of calluses and rough heels, and much preferred the fluid sound of monocular diplopia. Then he washed his hands of me and told me to see a corneal specialist.

Before being examined by the corneal doctor I was prepped by his technician, who took my medical history and carried out preliminary examinations. Agitated by my long search for a diagnosis, I told him about my many months with monocular diplopia.

"Anyone ever look under your eyelid?" he asked me.

"No, never," I said.

Another technician repeated the corneal topography on both my eyes. Then I went into the examining room of the corneal man, who reconfirmed the anomalous bulges and the diagnosis of keratoconus, though he admitted that it was a little peculiar that the bulge was on the nasal side of the cornea: gravity would have demanded a bulge at the bottom. The short-term solution was to wear a hard glass contact lens. Tears would fill the gap between the outside of my distorted cornea and the inside of the rigid glass lens, thereby restoring a perfectly spherical surface for light to enter. If it worked, it would also confirm the diagnosis. The longer-term solution, since the cornea would progressively thin and weaken, was either a corneal transplant or the implanting of a scaffolding of plastic circular struts into the cornea to support it.

Before I left I told my corneal specialist what his technician had said about looking under my left eyelid. He shrugged. A week later I went back to his office to be fitted for contacts. Agitated as usual, I told a different technician my whole story as she prepared to try a hard contact on me to see if it improved my vision. I also told her what the technician had said.

"I can look under your eyelid," she said. She had me lean back and she rolled up my eyelid like a window blind.

"There's a chalazion there," she said. That's a fancy word for a tiny hard lump on the inside of the upper eyelid, a sort of chronic stye. I didn't like the word *chalazion* either; it reminded me of both a venereal disease and a Spanish pork sausage. "It pushes on the cornea every time you blink or close your eyes, like someone sticking a finger into a balloon, and distorts your vision," she explained.

Retina, brain, lens, keratoconus. Everyone I consulted had tried, within and sometimes outside the confines of his specialty, to find some explanation for my symptoms, ignoring the obvious. It took a technician with no preconceptions to offer a commonsense explanation. My specialists were quite unabashed at their misdiagnoses. Because I had had a retinal problem in the past, everyone, myself included, refused to imagine any other cause, especially a simple, mechanical, and easily observable cause, like pressure on the cornea from a bump on the inner eyelid.

I tried the conservative treatment for the chalazion first: hot compresses and antibiotic creams to soften up and shrink the bump. But it had been around too long, and so a few weeks later another specialist cut it out and I've been okay since. Which leads me to the following section.

MAKING THE UNCONSCIOUS CONSCIOUS AGAIN

> We cannot be forever examining our foundations; we look particularly to those places where it is reported to us that they are insecure.
>
> —A. S. Eddington, *The Mathematical Theory of Relativity*

We become expert at the models and theories we use, unconscious of how we use them. This unconscious grasp then serves as a framework for further conscious advancement. One progresses from grasping an object to pitching a baseball to juggling, each accomplishment starting out as a conscious attempt and ending up as unconscious mastery. Conscious and unconscious play leapfrog, the conscious vaulting over the unconscious to then serve as a new unconscious foundation itself.

Faced with crises, the unconscious must become conscious again. When models produce paradoxes or conflicts, it becomes necessary to expose the taken-for-granted assumptions, *reculer pour mieux sauter*. That's what my clever eye technician did. When I later asked one of the doctors I had visited why he had never thought of looking for a bump on my eyelid, he told me that he had assumed that a bump large enough to dent my cornea every time I blinked would have been visible from the outside. It wasn't true. Only the technician was able to hear the facts as I explained them.

Einstein exposed the unconscious when he examined the foundations of classical physics. Physicists before him had unthinkingly assumed that simultaneity was a self-evident notion: to say that two events happened at the same time seemed a simple thing. Einstein made explicit how one would confirm that two events happen at the same time. This reexamination of fundamentals led him to the Theory of Special Relativity, whereby he discovered that, if the speed of light is the same for all observers, then length and time themselves must vary from one observer to another. When you say it like that, it sounds obvious: How can the speed of light possibly be the same for everyone no matter what speed each observer is traveling at, unless length and time themselves change with motion? It wasn't obvious before Einstein.

When the unconscious assumptions of everyday living begin to conflict with each other, it's time to bring them to the surface. Psychoanalysis aims to make the unconscious visible by talking and introspection. Tibetan Buddhists try to achieve the same result by observing the thoughts bubbling out of the mind. Dropping back is sometimes a good idea. The revision of fundamentals often marks great leaps forward.

ADDENDUM: GOETHE ON SYMBOLISM

I recently came across some remarks by Goethe in an essay on symbolism that reflect on the limitations of words and metaphors in approaching what he calls nature's inner relationships:

> Neither things nor ourselves find full expression in our words.
>
> Something like a new world is created through language, one consisting of the essential and the incidental.
>
> *Verba valent sicut numi.*[18] But there are different sorts of money: gold, silver, and copper coins, or paper money. The coins are real to a degree; the paper money is only convention.
>
> We get by in life with our everyday language, for we describe only superficial relationships. The instant we speak of deeper relationships, another language springs up: poetic language.
>
> In speaking of nature's inner relationships, we need many modes of description. I will mention four here:

Symbols

1. which are *physically*[19] and really identical with the object: e.g., we have learned to express magnetic effects, and now apply this terminology to related phenomena;
2. which are *aesthetically*[19] and really identical with the object. All good metaphors belong in this category, but we must guard against a display of wit which seems to relate the unrelated instead of finding true relationships;
3. which express a connection which is somewhat arbitrary rather than fully intrinsic; such a symbol, however, points to an inner relationship between phenomena. I would say these symbols are mnemonic in a higher sense, for ordinary mnemonics uses wholly arbitrary notation;
4. which are derived from mathematics. Because they are founded on intuitive perceptions, they can become identical with the phenomenon in the highest sense of the word.

We find instances of the first three symbols in language:

1. when, for example, the word expresses a sound (like the noun bang).
2. when the sound expresses an identical feeling (this often happens in inflected forms: banging).
3. when related words have a similar sound (like mine and thine); such words might be dissimilar (I and thou), but moi and toi are related in this way.

The fourth type, based on intuitive perceptions alone, cannot occur in language.